Q. No. M 542 / 070

B. Tech / Odd
(17-18) / Reg

2017-18

LINEAR ALGEBRA
(Open Elective)
M 542

Full Marks : 70 *Time* : 3 hours

The figures in the margin indicate full marks.

Answer should be brief and to the point.

Answer Q. No. 1 and any *four* from the rest.

14 + (14 × 4)

1. (*a*) Prove or give a counterexample: composition of two linear transformations commutes. 4

(*b*) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator given by

$$T((x, y)) = (x - y, x + y), \forall (x, y) \in \mathbb{R}^2$$

Check whether T is invertible or not. If T is invertible, then find the standard matrices for T and T^{-1}. 2 + 6

(*c*) Give an example of a system of linear equations with no solutions. 2

2. (*a*) Let $M_2(\mathbb{R})$ be an IPS with respect to the inner product $\langle A, B \rangle = tr(B^t A)$ for all $A, B \in M_2(\mathbb{R})$. Define the norm of a matrix A with respect to the above inner product. What is the norm of $A = \begin{pmatrix} 5 & -3 \\ -5 & 1 \end{pmatrix}$? Are $B = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 6 & 3 \\ 3 & 27 \end{pmatrix}$ orthogonal ? Justify it. Show that

$\{B, C\}$ can't be a basis for $M_2(\mathbb{R})$. 1 + 2 + 1 + 2 + 2

(b) Consider $S = \{(1, 0\ 1), (0, 1, 2), (0, 1, 1)\} \subset \mathbb{R}^3$ with respect to the standard inner product. Can you find an orthonormal basis for $\mathbb{R}^3$ from here ? If yes, find it; otherwise justify your answer. 6

3. (a) State and prove **Dimension theorem.** Verify it for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T((x, y, z)) = (x + 2y - z, y + z, x + y - 2z)$$

for all $(x, y, z) \in \mathbb{R}^3$. 1 + 6 + 4

(b) Find a basis for the vector space $M_n(\mathbb{R})$ of all $n \times n$ real matrices. 3

4. (a) Let $P_3(\mathbb{R})$ be the IPS with usual inner product generated by multiplying coefficients. Let $\{p, q\} \subset P_3(\mathbb{R})$ be an orthonormal set. Find $\|p + q\|$. 3

(b) Give an example (with justification) of a matrix $A \in M_3(\mathbb{R})$ which can't be diagonable over any field. 6

(c) Show that $P_n(\mathbb{R})$ and $\mathbb{R}^{n+1}$ are *isomorphic* linear spaces by presenting an explicit *isomorphism.* 5

5. (a) Solve the following system of equations using Gauss-Jordan elimination method:

$$x + 2y + z - 4w = 1$$

$$x + 3y + 7z + 2w = 2$$

$$x - 11z - 16w = -1.$$

7

(b) Show that the solution space of the following system of linear equations

$$x + 2y + 2z = 0 = x + y + 5z$$

forms a subspace of $\mathbb{R}^3$ over $\mathbb{R}$. Also, find a basis for it and the dimension of it. 3 + 3 + 1

6. (a) Solve the following system of linear equations:

$$x_1 + x_2 + x_3 = 15$$

$$x_4 + x_5 + x_6 = 6$$

$$x_1 + x_4 = 8$$

$$x_2 + x_5 = 7$$

$$x_3 + x_6 = 6.$$

8

(b) The images of unit vectors under the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ are given by

$$T((1,0)) = (2, 1, h) \text{ and } T((0,1)) = (3, k, 0).$$

Determine all the values of the parameters h and k for which T is one-one. 6

7. (a) Can the vector (3,1,1) be expressed as a linear combination of the vectors (2,5,–1), (1,6,0), (5,2,–4) ? Justify your answer. 4

(b) Let $S = \{(x, y, z) \in \mathbb{R}^3 \mid xy = z\}$. is S a subspace of $\mathbb{R}^3$ over $\mathbb{R}$? 3

(c) Obtain the Jordan canonical form of the following matrix

$$A = \begin{pmatrix} -3 & -2 & 5 & 3 \\ -1 & 0 & 1 & 2 \\ -4 & -3 & 6 & 4 \\ -1 & -1 & 1 & 3 \end{pmatrix}$$

by directly finding the eigen vectors and then find a basis which will yield the Jordan form. 7

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1ST SEM 20/11/2017

Q. No. MA - 01 / 017

B. Tech./Odd

2017-18/Reg

2017-18

MATHEMATICS - I

MA - 01

Full Marks : 70 Time : Three Hours

The figures in the margin indicate full marks.

Answer Question No. 1 and any *five* from rest.

1. (a) If $u = \log(\tan x + \tan y)$ then

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = \underline{\qquad}.$$ 3

(b) Test the convergence of the sequence $\{u_n\}$ where $u_n = 2(-1)^n$. 3

(c) Find the points of inflexion of the curve $x = (\log y)^3$. 4

2. (a) Find the asymptotes of the following curve

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$$

(b) Find the radius of curvature of following curve

$r = ae^{\theta \cot \alpha}$ at any point θ. 6+6

P.T.O.

3. (a) Apply mean value theorem and prove that

$$x > \sin x > x - \frac{1}{6}x^3; \quad 0 < x < \frac{x}{2}$$

(b) Using MVT prove that $\sqrt{101}$ lies between 10 and 10.05.

6+6

4. (a) Show that the function

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous and possesses partial derivatives at (0, 0) but is not differentiable at (0, 0).

(b) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xf\frac{y}{x} \text{ and}$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0.$$

6+6

5. (a) Prove that the function

$f(x, y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has neither a maximum nor a minimum at the origin.

(b) If $u = x^2 + y^2$, $v = x^2 - y^2$ and $x = r\theta$, $y = r + \theta$ the find the value of the Jacobian $\frac{\partial(u, v)}{\partial(r, \theta)}$.

6+6

6. (a) Test the convergence of the following series :

$$\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}.$$

(b) $\frac{x}{1} + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{x^7}{7} + \ldots\infty \left(x > 0, \; x \neq 1\right)$

6+6

7. (a) Test the convergence of the following series :

$$\frac{1}{a} - \frac{1}{a+b} + \frac{1}{a+2b} - \frac{1}{a+3b} + \ldots \quad a > 0, \; b > 0.$$

(b) Reduce the equation

$x^2 - 6xy + 9y^2 + 4x - 12y + 4 = 0$ to the canonical form and determine the type of the conic represented by it.

6+6

8. (a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{-1}=\frac{y}{2}=\frac{z}{1}$ and whose guiding curve is the ellipse $x^2+2y^2=1,\ z=3$.

(b) Find the equation of the cone whose vertex is the point (–1, 1, 1) and whose guiding curve is $3x^2-y^2=1,\ z=0$. 6+6

21.11.17

Q. No. MA - 331 / 706

B. Tech./Odd

2017-18/Reg

2017-18

MATHEMATICS - III

MA - 331

Full Marks : 70 Time : Three Hours

The figures in the margin indicate full marks.

The given symbols have their usual meanings.

Answer Question No. 1 and any *five* from the rest.

10+(12×5)

1. (a) Expand $f(z) = \dfrac{e^{2z}}{(z-1)^3}$ about $z = 1$ as a Laurent's series. Also indicate the region of convergence of the series. 3

(b) Form a PDE by eliminating the arbitrary function ϕ from $\phi\left(x+y+z, x^2+y^2+z^2\right) = 0$. 3

(c) Evaluate $\iint_s \vec{r} \cdot \hat{n} dS$,

where S is any closed surface, $\vec{r}$ is the position vector and $\hat{n}$ is unit outward normal vector to the surface. Hence evaluate the same taking S as the surface of the sphere $x^2 + y^2 + z^2 = 9$. 4

P.T.O.

2. (a) Show that the mapping $w=\sqrt{z}$ transforms the family of circles $|z-1|=c$ into the family of lemniscates $|w-1|+|w+1|=c$. 6

(b) Evaluate $\int_C \frac{z+1}{z^2+2z+4}\,dz$, where C is the circle $|z+1+i|=2$. 6

3. (a) State Cauchy's residue theorem. Evaluate $\int_C \frac{z^3-2z+1}{(z-i)^2}\,dz$, where C is the circle $|z|=2$.

2+4

(b) Find the analytic function $f(z)=u(x,y)+iv(x,y)$ if $u+v=\frac{\sin 2x}{\cosh 2y-\cos 2x}$. 6

4. (a) Find the Fourier series of the function

$$F(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

Hence show that $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}$. 4+2

(b) State a sufficient condition for the convergence of a Fourier series generated by a function. Find a Fourier series to represent x^2 in the interval $(-l,l)$. 2+4

5. (a) Solve the PDE : $\left(D^2-4D'^2\right)z=\frac{4x}{y^2}-\frac{y}{z^2}$. 6

(b) Obtain the canonical form of the PDE : $z_{xx}+x^2z_{yy}=0$. 6

6. (a) Find the general solution of the PDE: $xy^2p+y^3q=zxy^2-4x^3$. 6

(b) A spherical hole of radius b is made centrally through a sphere of radius a ($a>b$). Find the volume of the remaining sphere using double integration. 6

7. (a) Change the order of integration in

$\int_0^3\int_1^{\sqrt{4-y}}(x+y)\,dx\,dy$ and hence evaluate it. 6

(b) Evaluate

$$\iiint_W e^{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}\,dx\,dy\,dz$$

where $W=\left\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2\le 1\right\}$. 6

8. (a) State the physical interpretation of the divergence of velocity vector for a fluid. 2

(b) Using Green's theorem evaluate the integration $\int_C\left(x^2-xy^3\right)dx+\left(y^2-2xy\right)dy$ where C is the square with vertices (0,0), (2,0), (2, 2) and (0,2). 4

P.T.O.

(c) Apply Stokes' theorem for $\overrightarrow{F} = (y, z, x)$ to evaluate $\int_C \overrightarrow{F} \cdot \overrightarrow{ds}$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. 6

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Q. No. MAC - 01 /

B. Tech./Odd

2017-18/Reg

2017-18

MATHEMATICS - I

MAC - 01

Full Marks : 50 | Time : Three Hours

The figures in the margin indicate full marks.

Answer Question No. 1 and any *four* from rest.

1. (a) Find the point(s) of inflection of the curve

$$y = \frac{x^3}{a^2 + x^2}.$$ 3

(b) Suppose $y_1(t) = 5t^2$ and $y_2(t) = 3e^{-t}$ are two solutions of a second order homogeneous linear equation

$$\frac{d^2y}{dt^2} + p(t)\frac{dy}{dt} + q(t)y = 0.$$

Will the set $\{y_1, y_2\}$ form a set of fundamental solutions of this equation? Justify your answer. 4

(c) Find the radius of curvature at the point $\theta = \frac{\pi}{4}$ of the curve $x = a\cos^3\theta$ and $y = a\sin^3\theta$. 3

P.T.O.

2. (a) Find the asymptotes of the curve

$$3x^3+2x^2y-7xy^2+2y^3-14xy+7y^2+4x+5y=0.$$

(b) Apply mean value theorem to prove that

$$x>\log(1+x)>x-\frac{x^2}{2},\ x>0.$$

5+5

3. (a) If $u=\dfrac{(x^2+y^2)^n}{2n(2n-1)}+xf\left(\dfrac{y}{x}\right)+g\left(\dfrac{y}{x}\right)$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(x^2+y^2)^n.$$

(b) Discuss the continuity of the function

$$f(x, y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y)\neq(0,0),\\ 0 & \text{if } (x, y)=(0,0),\end{cases}$$

at (0, 0).

5+5

4. (a) The temperature T at any point (x, y, z) in space is given by $T=400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.

(b) Find a Fourier series representation of the function $f(x)$ on $-\pi\le x\le\pi$, where

$$f(x)=\begin{cases}-\pi, & -\pi<x<0,\\ x, & 0\le x\le\pi,\end{cases}$$

and $f(x+2\pi)=f(x)$. Hence deduce that

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots$$

5+5

5. Examine the convergence of the following infinite series :

(a) $\sum_{n=1}^{\infty}\dfrac{[(n+1)x]^n}{n^{n+1}}$; $x>0$

(b) $1+\dfrac{1}{2}+\dfrac{1.3}{2.4}+\dfrac{1.3.5}{2.4.6}+\ldots$

5+5

6. (a) Solve the differential equation

$$(D^2-5D+6)y=x^2e^{3x},\ \text{where } D\equiv\frac{d}{dx}.$$

(b) Solve the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\sin x$$

by the method of variation of parameters.

5+5

7. (a) Solve the following simultaneous ordinary differential equations :

$$\frac{dx}{x\left(y^2-z^2\right)}=\frac{dy}{y\left(z^2-x^2\right)}=\frac{dz}{z\left(x^2-y^2\right)}.$$

(b) If u, v and w are the roots of the equation

$(\alpha-x)^3+(\alpha-y)^3+(\alpha-z)^3=0$ in α, then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. 5+5

$z = \int e^{-2x} \cdot \sin x$

$= \sin x \cdot \frac{e^{-2x}}{-2} - \int \cos x \cdot \frac{e^{-2x}}{-2}$

$u = \int e^{-x} \sin x$

$u = \sin x \cdot \frac{e^{-x}}{-1} - \int \cos x \cdot \frac{e^{-x}}{-1}$

$= -\sin x \frac{e^{-x}}{1} + \int e^{-x} \cos x$

$= -e^{-x} \sin x + \cos x \cdot \frac{e^{-x}}{-1} - \int -\sin x \cdot \frac{e^{-x}}{-1}$

$= -e^{-x} \sin x - \cos x e^{-x} - f(x)$

$f(x) = \frac{-e^{-x} \sin x - e^{-x} \cos x}{2}$