10.09.18 (2nd Half)

300 B.Tech./Odd/Mid-Term/2018-19

2018-19

National Institute of Technology Durgapur

Mathematics-I

MAC 01

Full Marks: 30 Symbols have their usual meanings. Time: 2 Hours

Answer any SIX questions. 5×6

1. Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$ for $0 < x < \frac{\pi}{2}$.
2. If ρ_1, ρ_2 be the radii of curvature at the extremities of any chord of the cardiode $r = a(1 + \cos\theta)$, which passes through the pole, then prove that $\rho_1^2 + \rho_2^2 = \frac{16}{9}a^2$.
3. Find the asymptotes of the curve $x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$.
4. Evaluate
$$\iint_D e^{-x-y}\, dxdy,$$
where D is the interior of the triangle with vertices $(0,0), (1,3)$ and $(2,3)$.
5. Apply mean value theorem of appropriate order to prove that
$$x > \log(1+x) > x - \frac{x^2}{2}, \ x > 0.$$
6. Evaluate the triple integral
$$\iiint_V \frac{1}{(x+y+z+1)^3}\, dx\, dy\, dz$$
over the closed region V formed by the planes $x = 0$, $y = 0$, $z = 0$ and $x+y+z = 1$.
7. Evaluate the double integral
$$\iint_R (1 - \frac{x^2}{a^2} - \frac{y^2}{b^2})\, dx\, dy,$$
where $R = \{(x,y) : \ x \geq 0, \ y \geq 0, \ \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\}$.
8. (a) Evaluate
$$\int_0^\infty e^{-t^2}\, dt.$$
(b) Express
$$\int_0^1 t^m(1-t^n)^p\, dt$$
in terms of Beta function and hence evaluate $\int_0^1 t^4(1-t^3)^7 dt$.

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Samarjit Kar
21/8/18

10/09/18 (1st Half)

711

B.Tech/Odd/Mid-Term/2018 - 19

National Institute of Technology Durgapur

Mathematics-III

MAC - 331

Full Marks: 30 Symbols have their usual meanings. Time: 2 Hours

Answer any SIX. 5× 6

1. Solve the Lagrange's equation

$$\cos(x+y)p + \sin(x+y)q = z.$$

2. Find the general solution of the partial differential equation

$$(D^2 + DD' - 6D'^2)z = y\sin x.$$

3. Form a partial differential equation by eliminating the arbitrary functions f and g from $z = yf(x) + xg(y)$.

4. Let

$$f(z) = \begin{cases} e^{-\frac{1}{z^4}} \text{ if } z \neq 0; \\ 0 \text{ if } z = 0. \end{cases}$$

Show that

a) $f(z)$ is analytic everywhere except at $z = 0$;

b) $f(z)$ is not continuous at $z = 0$.

5. If $f(z) = u + iv$ is an analytic function of $z\ (= x + iy)$ and $u - v = e^x(\cos y - \sin y)$, find $f(z)$.

6. The population in a village in different years is given in the following table.

$Year\ (x)$:	1961	1971	1981	1991	2001
$Population(in\ 100's)\ (y)$:	46	66	81	93	101

Estimate the population of the village in the year 1996 using appropriate interpolation formula and write down the result correct upto six significant digits.

Samarjit Kar
28/8/18

7. (a) A dairy firm has two milk plants with daily milk production of 6 million liters and 9 million liters respectively. Each day the firm must fulfill the needs of its three distribution centers (A, B and C) which have milk requirement of 7, 5 and 3 million liters respectively. Cost of shipping one million liters of milk from each plant to each distribution center (DC) is given, in hundreds of rupees below.

		DC			
		A	B	C	Supply
Plants	1	2	3	11	6
	2	1	9	6	9
	Demand	7	5	3	

Formulate a linear programming model to minimize the transportation cost.

(b) Find basic feasible solutions of the following system of equations

$$2x_1 + x_2 - x_3 = 2,$$
$$3x_1 + 2x_2 + x_3 = 3.$$

Identify a degenerate solution, if there is any.

8. Solve the following Linear Programming Problem graphically:

$$\begin{aligned} \text{Maximize} \quad & z = 5x_1 + 7x_2, \\ \text{subject to} \quad & x_1 + x_2 \leq 500, \\ & 2x_1 - x_2 \leq 0, \\ & x_1 \geq 100, \\ & \text{and } x_1, x_2 \geq 0. \end{aligned}$$

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28/8/18